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ANALYTICAL ESTIMATION OF THE OPTIMAL PV PANEL TILT BASED ON A CLEAR-SKY IRRADIANCE MODEL

ANALITIČNA OCENA OPTIMALNEGA NAGIBA PV PANELA NA PODLAGI MODELA PROUČEVANJA SONČNEGA SEVANJA PRI JASNEM NEBU

Elena Golubovska[1] , Biljana Citkuseva Dimitrovska[2], Roman Golubovski[3ℜ]

Keywords: PV panel tilt, optimal PV panel inclination, PV conversion efficiency, sun position model, clear-sky solar irradiance model

Abstract

PV panel tilt and sun tracking are crucial aspects of PV conversion efficiency. We propose an analytical methodology for estimation of the optimal PV panel tilt based on calculation of the sun`s position and the application of a clear-sky solar irradiance model. Our methodology outputs three angles referencing a geolocation and the moment of interest: the incidence angle q, the sun altitude a and the sun azimuth z. The irradiance model estimates the solar irradiation at a geolocation that can be used for PV conversion estimation based on specified tilt β. The moment PV power is used for calculation of the daily energy production, and the optimal β is identified in the tilt range of 0° to 90°. Seasonal division of the year is performed, and optimal seasonal tilt is estimated based on the maximally produced seasonal energy, tested with every corresponding β.

ℜ Corresponding author: Prof. Dr, Roman Golubovski, UKIM, FNSM, Arhimedova bb, Skopje, N.Macedonia, Tel.: +389 70 206 459, E-mail address: roman@pmf.ukim.mk

1 University Ss. Cyril and Methodius, Faculty of Computer Science and Engineering, Skopje, N.Macedonia

2 University Goce Delcev, Faculty of Electrical Engineering, Shtip, N.Macedonia

3 University Ss. Cyril and Methodius, Faculty of Natural Sciences and Mathematics, Skopje, N.Macedonia

The methodology is tested on four typical seasonal models - 12 months, 4 three-month quarters, 2 half-year seasons and a single optimal annual fixed β The preliminary simulations produced promising results consistent with the practical engineering implementations.

Povzetek

Nagib PV-panela in sledenje soncu sta ključna vidika učinkovitosti PV-pretvorbe. Predlagamo analitično metodologijo za oceno optimalnega nagiba fotonapetostne plošče na podlagi izračuna položaja sonca in uporabe modela sončnega sevanja pri jasnem nebu. Naša metodologija prikaže tri kote, ki se nanašajo na geolokacijo in trenutek opazovanja: vpadni kot θ, višinski kot sonca a in azimut sonca z. Model obsevanja ocenjuje sončno sevanje na geolokaciji, ki se lahko uporabi za oceno pretvorbe PV na podlagi določenega nagiba β. To je trenutek, ko se PV moč uporabi za izračun dnevne proizvodnje energije, optimalni β pa se določi v območju nagiba od 0° do 90°. Izvede se sezonska delitev leta in optimalni sezonski nagib se oceni na podlagi največje proizvedene sezonske energije, testirane z vsakim ustreznim β Metodologija je preizkušena na štirih tipičnih sezonskih modelih – 12 mesecih, štirih trimesečnih kvartalih, dveh polletnih sezonah in enem optimalnem letnem fiksnem β. Predhodne simulacije dajejo obetavne rezultate, skladne s praktičnimi inženirskimi izvedbami.

1 INTRODUCTION

Renewable energies are subject to continuous research for their sustainability, contrary to the depletive and hazardous properties of fossil fuels and nuclear fission. Solar energy is obviously the most sustainable form, independent of other circumstances, until it reaches the atmosphere and is degraded acceptably while propagating through it. Solar irradiation is used efficiently by photovoltaic (PV) conversion, which is the cheapest electrical energy production technology compared to the rest. PV technologies are also affordable at the household level, making them globally popular today. The widespread market sustains a growing PV production industry that increases the PV conversion efficiency continuously and lowers the market costs. However, besides the improved material performances, planners of PV plants also tackle installation efficiency issues for maximizing energy production against lower costs. Among other things, they aim for optimal latitude (L) placement, as well as optimal panel tilt (β) for the sun`s incidence angle (θ) closest to the possible zenith, and longer possible under daylight. The incidence angle θ can be maintained optimally by horizontal azimuth tracking, but the panel azimuth is usually fixed towards local noon (1200h). The panel tilt optimization is subject to vertical inclination adjustment strategies, ranging from fixed tilt throughout the year to daily tracking (involving the use of computers equipped with sensors and actuators, introducing additional costs such as hardware, cabling, maintenance and energy consumption), depending on the economic circumstances. This paper proposes an analytical methodology for estimation of the optimal PV panel tilt based on the estimated sun position defined by the incidence angle θ, the sun`s altitude α and its azimuth z, and application of a clear-sky solar irradiance model. The sun's location is determined [1] against a specified geolocation for a specified date and time. The solar irradiance model [2], unlike efforts estimating locally arrived irradiation based on statistical meteorological data or measurement-based modelling [3] - [15], calculates the maximal possible incoming solar irradiation that can be used for PV conversion estimation considering the latitude placement and sun`s position, as well as the panel tilt β. This allows for momentary power estimation and possible energy production over a specified period. The methodology allows tracking optimal β on a daily basis (the daily fixed tilt for which maximal daily energy can be produced), or for arbitrary defined seasons (the fixed tilt for which maximal seasonal energy can be produced). This approach is tested on four typical seasonal models - 12 months, 4 three-month quarters, 2 half-year seasons, and a single optimal annual fixed β.

2 SUN POSITION MODEL

The current sun position model calculates the incidence angle θ that the sun`s rays fall under at a specific geolocation (latitude and longitude) in a specific moment of the year (date and time), being the angle between the sun`s ray falling on that location and its perpendicular vertical line, as well as the seasonal sun altitude α and the daily azimuth z. In order to calculate these three essential angles, additional specifics regarding the earth's rotations need to be considered. Figure 1 shows the earth's annual (365.25 days) rotation around the sun, as well as the fixed declination δ of the earth's axis (23.45°), which oscillates with respect to the sun, producing on earth a solar declination angle between ± 23.45° depending on the moment in the year.

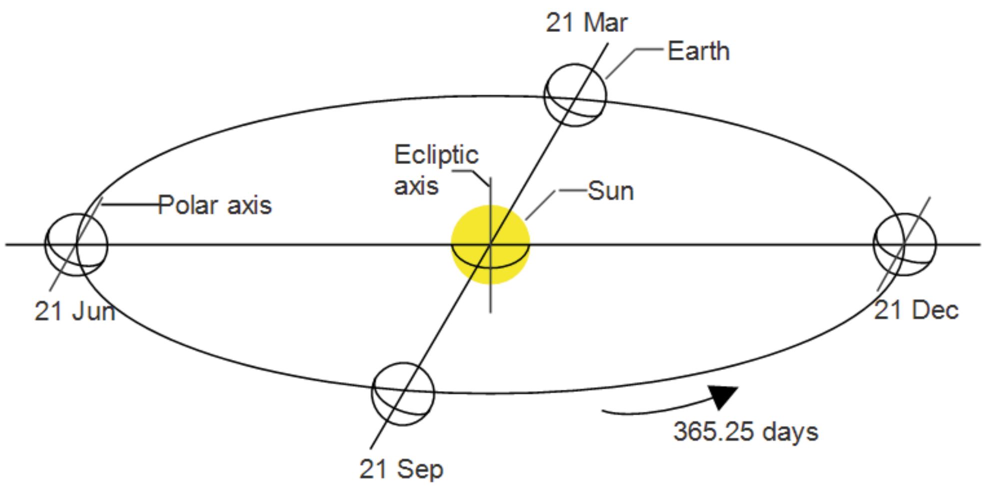


Figure 1: *Annual motion of the earth around the sun*

There are four fiducial dates during the year - 21 Jan (the summer solstice), which is the longest (summer) day in the northern hemisphere; 21 Dec (the winter solstice), which is the shortest (winter) day in the northern hemisphere; and 21 Mar and 21 Sep (the two equinoxes) with equal duration of their day and night. The daily declination δ for day N (of the 365 in a year) according to ASHRAE (the American Society of Heating, Refrigerating and Air-Conditioning Engineers) can be calculated with the expression 2.1.

$$\delta = 23.45° \, sin\left[\frac{360°}{365}(N-81)\right] \qquad [°] \qquad (2.1)$$

The specific moment is defined by the input date and time, expressed as the day N [1~365] in the year, and the local standard time (LST) expressed as local minute time (LMT) [min] in that day. The moment needs to be converted to a current angle with respect to a reference meridian. This means that the specified moment of time needs to be converted from LST to the local apparent solar time (AST), and then to the hour angle h representing that moment. In the local noon, LST should be exactly 1200h (midday), and so should the AST correspond to the solar zenith (1200h). However, during the annual rotation around the sun the earth's path varies, so does its speed around the sun. Due to some specifics, the rotational speed around its own axis also varies. These variations imply two corrections required for acceptable LST to AST conversion. The first one is the "equation of time" (EoT), which considers the eccentricity of the earth's orbit around the sun, and for day N is determined with the expression 2.2.

$$EoT = 9.87\sin(2B) - 7.53\cos(B) - 1.5\sin(B) \qquad [min] \qquad (2.2)$$

$$B = (N-1)\frac{360°}{364} \qquad [°]$$

The second correction is the "longitudinal correction" of LST, given in expression 2.3, which represents the current time with respect to a global time zone (T_GMT) or a standard meridian (SM) of the 24 defined for zoning the globe. It takes 4min for the sun to traverse 1°, and LST is "constant" in the watch for the whole 15° (representing an hour from 24 zones). The correction is intended to cancel out the running difference between the local longitude (LL) and the SM. Additionally, the daylight saving (DS) can be considered, expressed in [min] (being 0 when ignored and 60 otherwise).

$$AST = LST + EoT \pm 4(SL - LL) - DS \qquad [°] \qquad (2.3)$$

Now, the current hour angle h considering the current AST is given in the expression 2.4:

$$h = (AST - 12)\cdot 15° \qquad [°] \qquad (2.4)$$

Having h correspond to the specified date and time, it can be put in the context of the corresponding latitude L, solar declination δ, local zenith Φ, sun`s altitude $\alpha$ and sun`s azimuth z. Figure 2 and Figure 3 depict the context in which the angle parameters correlate. The sun`s altitude α oscillates with the seasonal dynamics, and can be correlated trigonometrically with the local latitude L, the solar declination δ and the hour angle h in expression 2.5.

$$\alpha = sin^{-1}[sin(L)sin(\delta) + cos(L)cos(\delta)cos(h)] \qquad [°] \qquad (2.5)$$

The sun`s azimuth z oscillates with the daily dynamics, and can be correlated trigonometrically with the sun`s altitude α, the solar declination δ and the hour angle h in expression 2.6.

$$z = sin^{-1}\left[\frac{cos(\delta)sin(h)}{cos(\alpha)}\right] \qquad [°] \qquad (2.6)$$

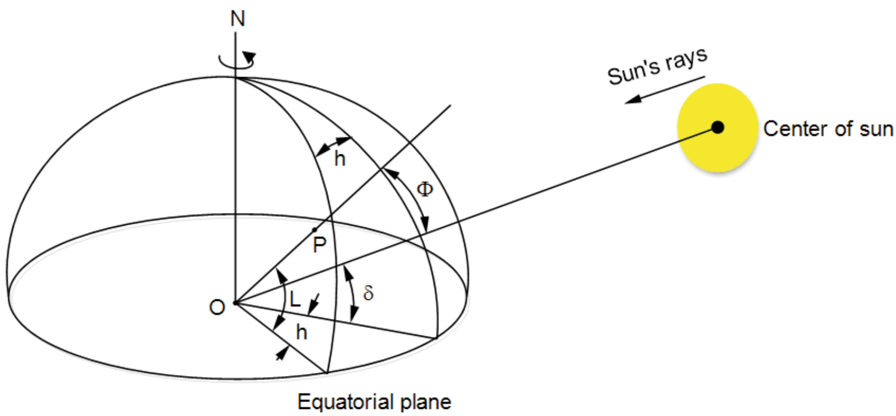


Figure 2: *Definition of latitude, hour angle and solar declination*

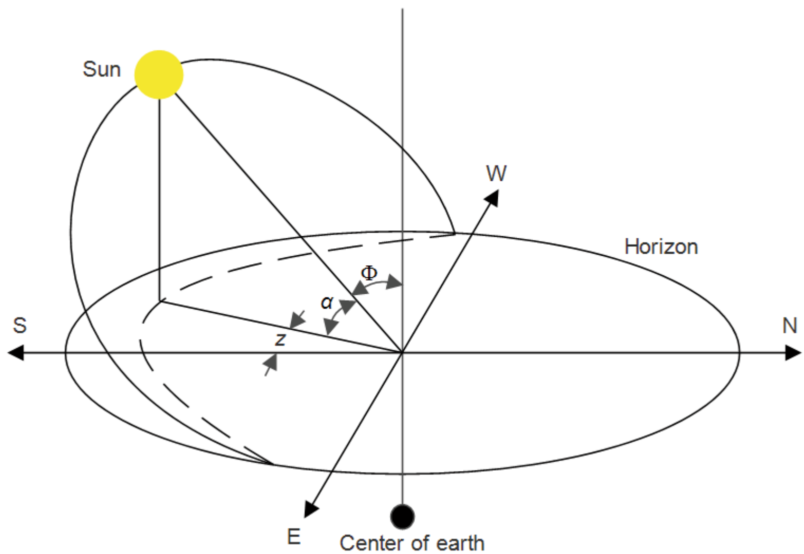


Figure 3: *Solar angles defining its position in the sky along its daily path*

Figure 4 defines the trigonometrical context of the inclined plane. The incident angle θ equals the zenith angle Φ if the plane is not tilted, but for a panel with tilt $\beta > 0°$ and own azimuth fixed toward local noon (0°), expression 2.7 gives the cos(θ) required by the PV conversion model.

$$\begin{aligned} cos(\theta) = {} & sin(L)sin(\delta)cos(\beta) - cos(L)sin(\delta)sin(\beta)cos(Z) + \\ & + cos(L)cos(\delta)cos(h)cos(\beta) + \\ & + sin(L)cos(\delta)cos(h)sin(\beta)cos(Z) + cos(\delta)sin(h)sin(\beta)sin(Z) \end{aligned} \tag{2.7}$$

3 ENERGY CONVERSION MODEL

The implemented PV energy conversion model is of the "clear-sky" type [2]. The clear-sky solar irradiation model assumes ideal meteorological conditions, where the irradiation of the location of interest is unobstructed by clouds or other atmospheric circumstances. Although not real, such an ideal context is optimal for comparative analysis. The extraterrestrial solar radiation arrives at the outer border of the earth's atmosphere in almost constant intensity, defined as the solar constant G_{SC}. The solar constant is the quantity of solar radiation arriving perpendicular to the atmosphere at an average distance from the sun, with very small variations between different analytical estimations and satellite measurements.

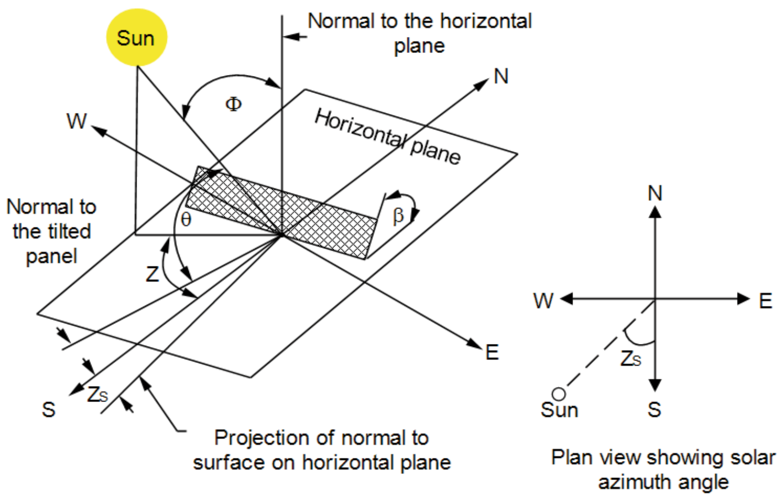


Figure 4: *Solar incidence angle for a non-horizontal surface*

The value used in this paper is 1367 [W/m^2], as proposed by Iqbal (1983) [16]. However, the sun's radiation declines intrinsically with an annual rate of 0.02% (insignificant, and can be disregarded), and varies $\pm$3.3% due to the earth's variable distance from the sun, which is significant and must be taken into consideration. This results in the effective arrived radiation at the atmosphere G_{ATM}, expressed for day N of the year according to Spencer (1971) [17] with the following formula:

$$G_{ATM} = G_{SC}\left[1 + 0.033\cos\left(\frac{360^\circ}{365}N\right)\right] \qquad [W/m^2] \qquad (3.1)$$

The direct radiation (beam radiation) G_{DIR} is the amount of irradiation propagated to the location of interest through the atmosphere without dissipation. Diffuse radiation (sky radiation) G_{DIF} is the dissipated irradiation arriving at the location of interest from the surrounding space. The total irradiation G is the amount measured at the location of interest. The energy model is expected to provide an analytical estimation of the PV conversion power, considering the direct and diffuse irradiation and ignoring the reflection component. The PV conversion efficiency η is assumed to be up to 20%, as the commercial PV technologies currently offer. The expression for the PV converted power at the location of interest is:

$$P = \eta \, [(G_{DIR} + G_{DIF}) \, S_{DIR} + G_{DIF} \, S_{DIF}] \qquad [W] \tag{3.2}$$

G_{DIR} and G_{DIF} result from the interaction of G_{ATM} with the atmosphere (its vapor molecules and microparticles) while propagating and dissipating through it. This interaction is defined as atmospheric transmittance τ. The transmittance of direct irradiation τ_{DIR} is proposed by Hottel (1976) [18] in the following expression:

$$\tau_{DIR} = a_0 \, r_0 + a_1 \, r_1 \, e^{-\frac{k \cdot r_k}{cos(\Phi)}} \tag{3.3}$$

where Φ is the zenith angle (90° - α), and a_0, a_1 and k are the atmospheric parameters of a clear sky with visibility up to 23 km, and for altitudes (A) of up to 2.5 km, given in expression 3.4:

$$a_0 = 0.4237 - 0.00821 \, (6 - A)^2 \tag{3.4}$$

$$a_1 = 0.5055 - 0.00595 \, (6.5 - A)^2$$

$$k = 0.2711 - 0.01858 \, (2 - A)^2$$

and where r_0, r_1 and r_k are the corrective climate factors declared in Table 1:

***Table 1:** Corrective climate factors*

Climate type	r_0	r_1	r_k
Tropical latitudes (0° ≤ L < 23.45°)	0.95	0.98	1.02
Mid latitudes (23.45° ≤ L < 66.55°) in summer	0.97	0.99	1.02
Mid latitudes (23.45° ≤ L < 66.55°) in winter	1.03	1.01	1.00
Polar latitudes (66.55° ≤ L ≤ 90°) in summer (during daylight)	0.99	0.99	1.01

According to the same model, the transmittance of diffuse radiation τ_{DIF} is given in expression 3.5:

$$\tau_{DIF} = 0.271 - 0.294 \, \tau_{DIR} \tag{3.5}$$

Knowing both transmittances, τ_{DIR} and τ_{DIF}, allows calculation of the direct and diffuse irradiation that fall perpendicular at the locations of interest, $G_{DIR(n)}$ and $G_{DIF(n)}$, as given in expression 3.6:

$$G_{DIR(n)} = \tau_{DIR} \, G_{ATM} \qquad [W/m^2] \tag{3.6}$$

$$G_{DIF(n)} = \tau_{DIF} \, G_{ATM} \qquad [W/m^2]$$

Knowing the tilt β, and having calculated cos(θ) it is now possible to correct both irradiation components in their final form:

$$G_{DIR} = G_{DIR(n)} \, cos(\theta) = G_{ATM} \, \tau_{DIR} \, cos(\theta) \qquad [W/m^2] \tag{3.7}$$

$$G_{DIF} = G_{DIF(n)} \, \frac{1+cos(\beta)}{2} = G_{ATM} \, \tau_{DIF} \, \frac{1+cos(\beta)}{2} \qquad [W/m^2] \tag{3.8}$$

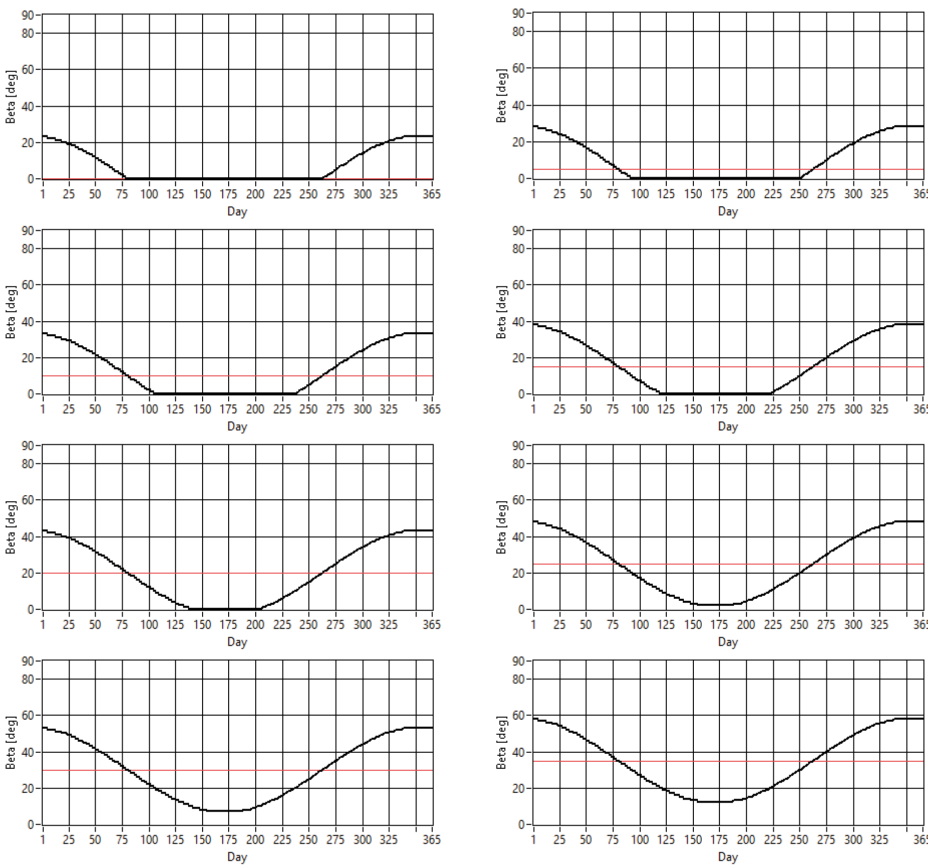
In expression 3.5 it is obvious that the diffuse component is neglective compared to the direct, so the final expression for the PV conversion power is:

$$P = \eta \, (G_{DIR} \, S_{DIR} + G_{DIF} \, S_{DIF}) \qquad [W] \tag{3.9}$$

Tracing expression 3.9 backwards, it is clear that PV conversion power P at a specified geolocation can be estimated with the clear-sky model and the sun position model for a specified moment in the year by just knowing the date and time.

4 OPTIMAL TILT CALCULATION

Optimal panel inclination is the tilt β for which the panel produces maximal daily energy DE_{MAX}, calculated by integrating its power P in 15min intervals during the daylight of a specified date (N). The algorithm for optimal tilt determination checks (calculates) DE against "all" β values in the range of 0° to 90° with a step of 1°. The tilt that produces DE_{MAX} is the optimal daily tilt β_{OD}. Figure 5 provides the optimal β curves for "all" latitudes in the north hemisphere from 0° (the equator) to 90° (the North Pole) with the step 5°. Every graph corresponds to a latitude labeled with a red horizontal line. Added is the latitude of Skopje (42°).



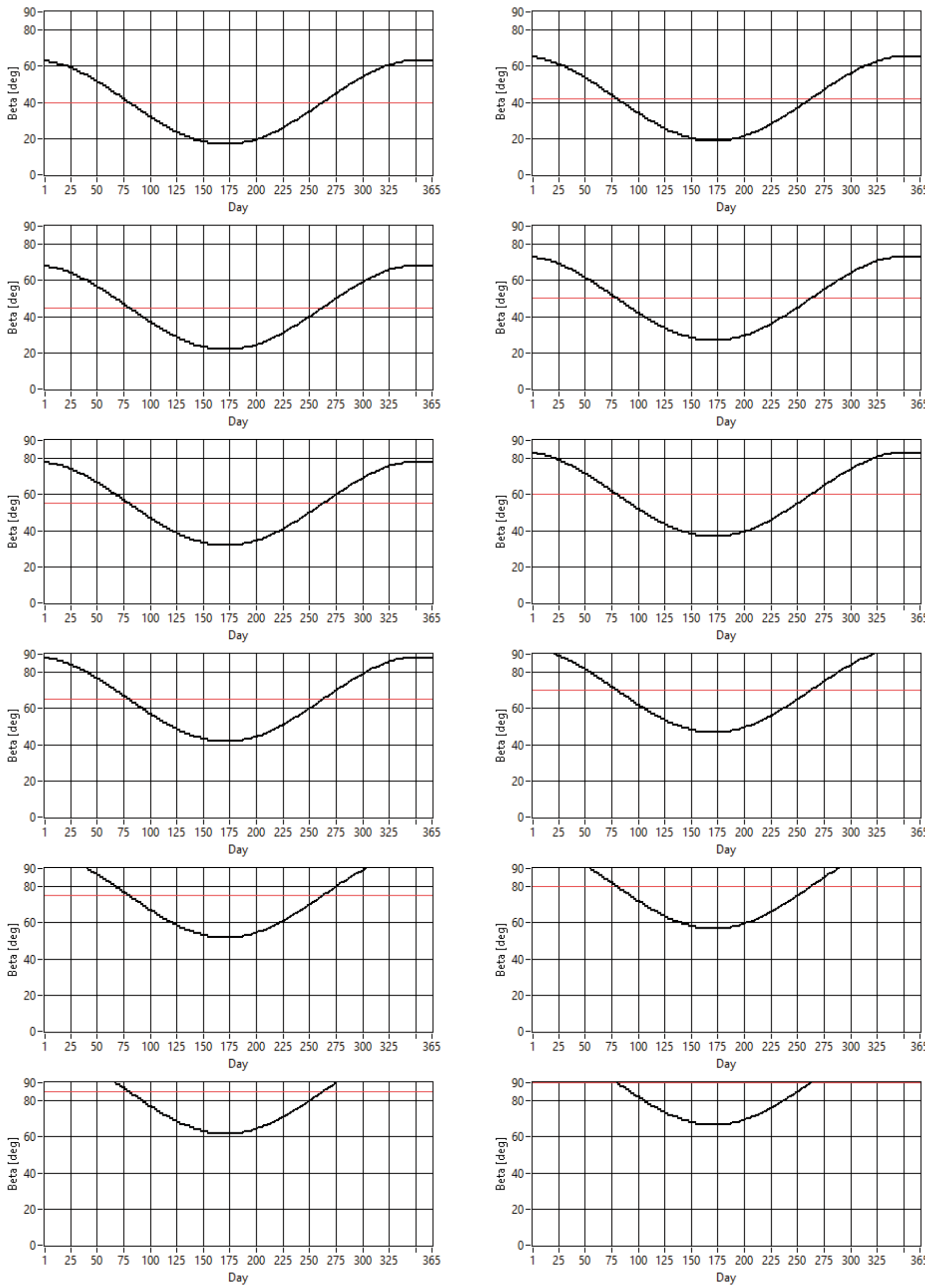


Figure 5: *Annual tracking of the daily optimal tilt for all north hemisphere latitudes from 0° to 90° with the step 5° (including 42°)*

It is obvious that the tropical curves (0° to 23.45°) are "cut below" at 0°, since, during summer in the northern hemisphere, these latitudes are closer to the zenith than the equator, and the panels are laid on the ground optimally and cannot tilt down anymore. In the mid-latitudes (from 23.45° to 66.55°) the optimal β oscillation of $\pm$ 23.45° is obvious, due to the corresponding oscillation of the solar declination δ. The polar latitudes (above 66.55°) are in daylight half the year also due to δ. Daily tracking (especially for large PV plants) requires the use of computerized equipment (for automatic tilting), that may not always be economically feasible, so the most widespread strategy is seasonal optimization. Being able to calculate optimal tilt β_{OD} for daily tracking allows the determination of seasonal optimal tilt β_{OS}. After an arbitrary season is defined with starting and ending dates, the cumulative seasonal energy SE is calculated for the whole season for every β_{OD} of the embraced dates. When SE_{MAX} is identified, its corresponding β_{OD} is the optimal seasonal tilt β_{OS}. The methodology was tested with four scenarios:

1. Monthly tilting - 12 seasons with optimal tilt
2. Four seasons - defined to have the least possible β_{OD} variation among the embraced days
   S#1 → 5 Nov ~ 4 Feb (winter, 92 days)
   S#2 → 5 Feb ~ 6 May (spring, 91 days)
   S#3 → 7 May ~ 5 Aug (summer, 92 days)
   S#4 → 6 Aug ~ 4 Nov (autumn, 90 days)
3. Two seasons - defined to have the least possible β_{OD} variation among the embraced days
   H#1 → 21 Sep ~ 20 Mar (winter, 181 days, 21 Dec - centered)
   H#2 → 21 Mar ~ 20 Sep (summer, 184 days, 21 Jun - centered)
4. Fixed annual tilt

Table 2 shows the results of the simulation of the four scenarios. The monthly tilting (scenario #1) for all latitudes is presented graphically in Figure 6.

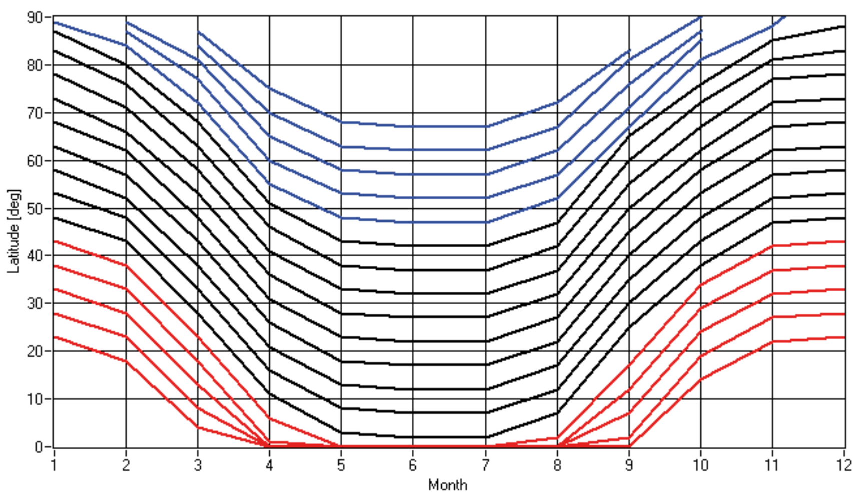


Figure 6: *Annual latitude curves of the monthly optimized tilt*

Table 2: *Seasonal tilt optimization*

Lat	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	S#1	S#2	S#3	S#4	H#1	H#2	365
0°	23	18	4	0	0	0	0	0	0	14	22	23	23	0	0	0	21	0	1
5°	28	23	8	0	0	0	0	0	2	19	27	28	28	5	0	5	26	0	5
10°	33	28	13	0	0	0	0	0	7	24	32	33	33	10	0	10	31	0	10
15°	38	33	18	1	0	0	0	0	12	29	37	38	38	15	0	15	35	0	14
20°	43	38	23	6	0	0	0	2	17	34	42	43	43	20	0	20	40	0	19
25°	48	43	28	11	3	2	2	7	25	38	47	48	48	26	2	25	45	3	25
30°	53	48	33	16	8	7	7	12	30	43	52	53	53	31	7	30	49	7	30
35°	58	52	38	21	13	12	12	17	35	48	57	58	58	36	12	35	54	12	35
40°	63	57	43	26	18	17	17	22	40	53	62	63	63	41	17	40	58	17	40
45°	68	62	48	31	23	22	22	27	45	58	67	68	68	46	22	45	63	22	45
50°	73	66	53	36	28	27	27	32	50	62	72	73	73	51	27	50	67	27	50
55°	78	71	58	41	33	32	32	37	55	67	77	78	78	56	32	55	71	32	55
60°	83	76	63	46	38	37	37	42	60	72	81	83	83	61	37	60	75	37	60
65°	87	80	68	51	43	42	42	47	65	76	85	88	87	66	42	65	78	42	65
70°	89	84	72	55	48	47	47	52	67	81	88	100	89	65	47	65	80	47	54
75°		87	77	60	53	52	52	57	71	85				68	52	68	82	52	56
80°		89	81	65	58	57	57	62	76	87				70	57	70	85	57	58
85°			84	70	63	62	62	67	81	90				72	62	72	88	62	62
90°			87	75	68	67	67	72	83					74	67	75		67	67

5 DISCUSSION OF THE RESULTS

As it can be seen in Table 2, the monthly (12 seasons) optimal tilting of the tropical and the mid-latitudes β_{OS} curves show the same oscillation of $\pm$ 23.45°, corresponding to the annual solar declination (δ) change. The zeros may correspond to either the optimal tilts, or to the minimal possible tilts when the panels lie on the ground and cannot tilt below, in a negative angle. In the polar latitudes, the optimal tilt can be calculated for the dates during the polar day of the Northern Hemisphere. There are no β_{OS} propositions for the polar latitudes that are entirely in the polar night. In the four-seasons model, the seasonal dates are chosen to have the least possible β_{OD} variation among the embraced days, using the graphs in Figure 5. Their β_{OS} are in accordance with the dates of all altitudes that have daylight. The same can be said for the two-seasons model. It is also obvious that fixed annual model β_{OS} for all tropical and mid-latitudes equals the corresponding latitudes, which confirms the standard engineering recommendation and practice. As expected, the polar latitudes have optimal tilting, based on the dates with daylight. The simulation results are consistent with common engineering practices, pending the experimental validation.

6 CONCLUSION

Solar energy is an abundant source of sustainable green energy, yet delicate from a harvesting perspective, due to the blocking properties of the unpredictable atmospheric conditions (vapor and particles), as well as the solar incidence angle on which the PV conversion efficiency depends.

Furthermore, the more directly (perpendicular to the sun`s rays) the PV panels are exposed, the more heated they become, which, in turn, lowers the PV conversion efficiency. Continuous automated tracking (at least for the optimal tilt, and preferably for the optimal azimuth too) is usually a significant expense, so efforts are made to maximize energy harvesting based on the optimal "seasonal" fixed tilt, with latitude being the fundamental parameter and "season" being a sequence of days with no significant change in energy conversion if the tilt is optimized daily. The problem of optimal fixed tilt calculation is recognized by the engineering community, and enormous effort is made worldwide to define such an analytical model for optimal tilting without expensive tracking or time-consuming measurements, which can be depicted in lots of published papers, like [3] - [15]. Some approaches use statistical meteorological data as a basis to determine the geometrical circumstances under which the panel had its optimal tilt for maximal efficiency. Others use regression over measured data under known weather circumstances to model the analytical expression for optimal tilt calculation, which, on the other hand, introduces some "off latitude" deviation in the results. Some of the authors consider the context's parameters, that have a significant impact on the measurements, like vapor, microparticles and pollution, which influence the local atmosphere over the panels. Interestingly, very few consider the operational heat, which lowers the PV conversion efficiency. If not addressed properly with cooling, it does coerce the model off the latitude value. Especially significant we find the work of Ogundimu @ al [14], providing a comprehensive overview of the contributions of previously published models proposing methodologies for optimal tilting at specific latitudes, providing correcting parameters that those scholars have calculated based on direct measurements or local (historical) statistical meteorological data. We believe that these approaches based on measured data or meteorological information do not always consider all circumstances influencing the resulting PV conversion efficiency, leading to those latitude corrections.

Our proposed concept for analytical estimation of the optimal PV panel tilt, based on the su`sn position and clear-sky irradiance models, provides a reliable and consistent methodology for daily tracking and arbitrary seasonal tilt optimization under ideal atmospheric conditions, as Nakamura @ al [15] support by their experimental setup. This approach is not burdened by costly and time-consuming measurements, heavy meteorological data statistics, or regression modeling. The algorithm is precise and easy to implement, thus providing an affordable and straight-forward-to-use tool for that purpose. If significant, local atmospheric conditions could be considered in the irradiance model, which would be one of the next development steps.

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