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AN APPROXIMATE MODEL FOR DETERMINING THE RESISTANCE OF A HEMISPHERICAL GROUND ELECTRODE PLACED AT THE TOP OF A NON-HOMOGENEOUS TRUNCATED CONE

PRIBLIŽNI MODEL ZA DOLOČANJE UPORA POLOKROGLE OZEMLJENE ELEKTRODE, NAMEŠČENE NA VRH NEHOMOGENEGA PRISEKANEGA STOŽCA

Dragan Vučković[1ℜ], Dejan Jovanović[2], Nenad Cvetković[2], Miodrag Stojanović[1], Dragan Tasić[1]

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Abstract

The procedure for analysis of a hemispherical ground electrode placed at the top of a hill is presented in the paper. The mountain is modelled as a truncated cone of finite height consisting of two homogeneous areas, each having different electrical characteristics. The applied approach is a combination of several recently proposed procedures. It includes the application of the Estimation method and the use of an approximate expression in closed form. The obtained results are validated and compared with those obtained using the COMSOL program package.

1ℜ Corresponding author: M.Sc Dragan Vučković, University of Nis, Faculty of Electronic Engineering, Department of Power Engineering, A. Medvedeva 14, 18000 Nis, Serbia, Tel.: +381 18 529 305, E-mail address: dragan.vuckovic@elfak.ni.ac.rs

1 University of Nis, Faculty of Electronic Engineering, Department of Power Engineering, A. Medvedeva 14, 18000 Nis, Serbia

2 University of Nis, Faculty of Electronic Engineering, Department of Theoretical Electrical Engineering, A. Medvedeva 14, 18000 Nis, Serbia

Povzetek

V prispevku je predstavljen postopek analize polkrogle ozemljitvene elektrode, nameščene na vrhu hriba. Gora je modelirana kot prisekan stožec končne višine, sestavljen iz dveh homogenih območij, od katerih ima vsako različne električne značilnosti. Uporabljeni pristop je kombinacija več nedavno predlaganih postopkov. Vključuje uporabo metode ocenjevanja in uporabo približnega izraza v zaprti obliki. Dobljene rezultate smo validirali in primerjali z rezultati, pridobljenimi s programskim paketom COMSOL.

1 INTRODUCTION

The most important feature of the grounding electrode, the resistance value, is conditioned by electrode geometry, conductor characteristics, soil structure and the physical shape of the surrounding ground. Various procedures are based on different numerical and semi-numerical methods, depending on the soil structure and corresponding geometrical model. For example, in *[1-2],* the ground is modelled as a homogeneous half space of a flat surface. Very often, the non-homogeneous ground is modelled as multi-layered *[3-4]*, sectoral *[5]*, a semi-spherically *[6-7]* or semi-cylindrically shaped domain *[8]*.

The problem of determining the resistance of a hemispherical ground electrode placed at the top of a non-homogeneous truncated cone modelled with three homogeneous domains having different electrical characteristics is investigated in this paper. The procedure applied in this paper is based on approaches from *[9-11]*. The procedure for analysis of a hemispherical ground electrode placed at the top of a hill was proposed in *[9]*. This approach was improved in *[10]* by assuming current density distribution in two different forms, depending on the observed domain. The method applied in this paper is an extended application of the approach based on procedures from *[9-10]* and presented in *[11]*. The previously mentioned approach offers the possibility of modelling a truncated cone as a non-homogeneous domain consisting of different homogeneous areas.

The applied procedure does not require integration, and all the used approximate expressions are given in closed form. The COMSOL program package based on the Finite Element Method was used to validate the obtained results.

2 PROBLEM DESCRIPTION

The hemispherical ground electrode is placed at the top of a non-homogeneous truncated cone consisting of three homogeneous domains having specific conductivities σ_1 , σ_3 and σ_3 respectively. The boundary surfaces between the domains are flat.

The radius of the electrode is r_1 and the cone base radius is d_1. The depth of the boundary surfaces is labelled with d_1 , and the cone slope is defined with angle α. In this paper, the proposed solution for the structure from Fig. 1 is based, as has already been emphasised, on the approaches from *[9-11]*. However, the chosen model of a non-homogeneous truncated cone is a more complex structure related to those from *[9-11]*. One could expect that, in general, the case ground structure is non-homogeneous, and therefore, the model from Figure 1 can be used for approximating ground non-homogeneity.

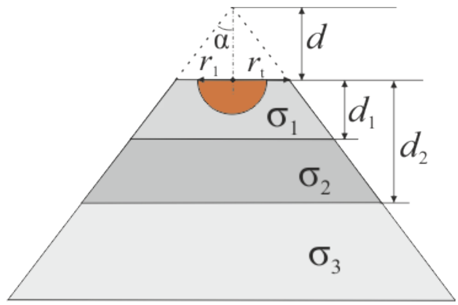

Figure 1: *The hemispherical ground electrode and truncated cone approximated with three homogeneous domains (flat boundary surface).*

3 PROCEDURE DESCRIPTION

The procedure for determination of the resistance of hemispherical ground electrode placed at the top of the truncated cone approximated with three homogeneous domains (Figure 2) will be derived, in order to generate the procedure for determination of the approximate resistance of the electrode from Figure 1. It is based on the approaches proposed in *[11]*. The boundary surfaces between domains are calotte surfaces having the radii r_3 and r_3, and the rest of the geometry parameters can be seen in Figure 2.

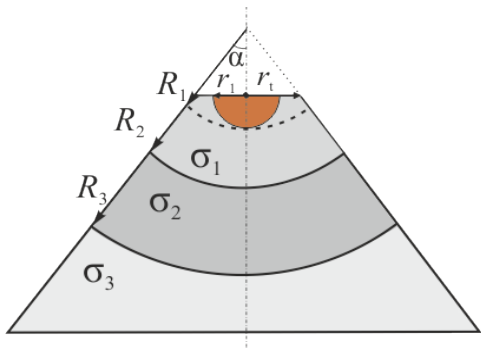

Figure 2: *The hemispherical ground electrode and truncated cone approximated with three homogeneous domains (oval boundary surface).*

Based on the procedure described in *[11]*, the current densities in the cone domain can be expressed as,

$$\vec{J}_1 = \frac{I}{2\pi r^2}\hat{r},\ r_1 < r < r_t \quad \text{, and} \tag{3.1a}$$

$$\vec{J}_2 = \frac{I}{2\pi(1-\cos\alpha)R^2}\hat{R},\ R_1 < R < \infty \tag{3.1b}$$

In previous expressions, r is a radial coordinate, while $\hat{r}$ is the corresponding spot of the coordinate system having origin at the middle point of the upper surface of the truncated cone. Similarly, $\hat{R}$ and $\hat{R}$ are the radial coordinate and corresponding spot of the coordinate system having origin at the imaginary top point of the cone *[11]*.

Using a local form of Ohm's law, for the electric field is obtained,

$$\vec{E}_{11} = \frac{\vec{J}_1}{\sigma_1} = \frac{I}{2\pi\sigma_1 r^2}\hat{r},\ r_1 < r < r_\mathrm{t} \tag{3.2a}$$

$$\vec{E}_{12} = \frac{\vec{J}_2}{\sigma_1} = \frac{I}{2\pi\sigma_1\left(1-\cos\alpha\right)R^2}\hat{R},\ R_1 < R < R_2 \tag{3.2b}$$

$$\vec{E}_{2} = \frac{\vec{J}_2}{\sigma_2} = \frac{I}{2\pi\sigma_2\left(1-\cos\alpha\right)R^2}\hat{R},\ R_2 < R < R_3 \tag{3.2c}$$

$$\vec{E}_{3} = \frac{\vec{J}_2}{\sigma_3} = \frac{I}{2\pi\sigma_3\left(1-\cos\alpha\right)R^2}\hat{R},\ R_3 < R < \infty \tag{3.2d}$$

The potential φ_s of the electrode surface from Fig 2 can be determined approximately as *[9, 12]*,

$$\varphi_\mathrm{s} = \int_{r_1}^{r_\mathrm{t}} E_{11}\,\mathrm{d}\,r + \int_{R_1}^{R_2} E_{12}\,\mathrm{d}\,r \int_{R_2}^{R_3} E_2\,\mathrm{d}\,R + \int_{R_3}^{\infty} E_3\,\mathrm{d}\,R \tag{3.3}$$

Now, for the potential φ_s obtains,

$$\begin{aligned}\varphi_\mathrm{s} = {} & \frac{I}{2\pi\sigma_1}\left[\frac{1}{r_1}-\frac{1}{r_\mathrm{t}}+\frac{1}{\left(1-\cos\alpha\right)}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right]+\frac{I}{2\pi\sigma_2}\frac{1}{\left(1-\cos\alpha\right)}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+ \\ & +\frac{I}{2\pi\sigma_3}\frac{1}{\left(1-\cos\alpha\right)}\frac{1}{R_3}.\end{aligned} \tag{3.4}$$

The electrode resistance value is

$$\begin{aligned}R_\mathrm{g} = \frac{\varphi_\mathrm{s}}{I} = {} & \frac{1}{2\pi\sigma_1}\left[\frac{1}{r_1}-\frac{1}{r_\mathrm{t}}+\frac{1}{\left(1-\cos\alpha\right)}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\right]+ \\ & +\frac{1}{2\pi\sigma_2}\frac{1}{\left(1-\cos\alpha\right)}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)+\frac{1}{2\pi\sigma_3}\frac{1}{\left(1-\cos\alpha\right)}\frac{1}{R_3}.\end{aligned} \tag{3.5}$$

There is interest to emphasise that in the previous expression

$$R_1 = \frac{r_\mathrm{t}}{\tan\alpha} + r_1 \tag{3.6}$$

4 ESTIMATION METHOD

In order to use the previously described procedure for the determination of the approximate resistance of the hemispherical grounding electrode from Figure 1, the Estimation method is introduced into the procedure.

From Figure 3 can be noticed the upper ($R_{2\mathrm{e}}$, $R_{3\mathrm{e}}$) and lower ($R_{3\mathrm{i}}$, $R_{3\mathrm{i}}$) values of the boundary surface radii. The approximate resistance of the system can be determined as the arithmetic mean of the resistance values calculated for the upper and lower radii values.

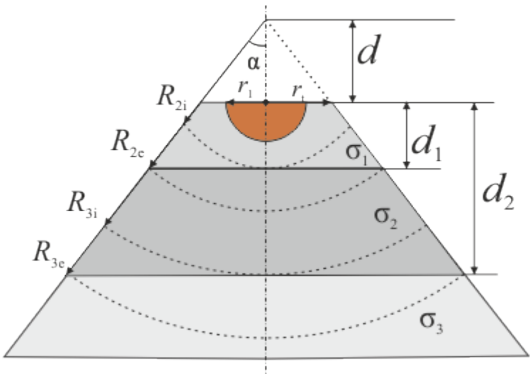

Figure 3: *Estimation method application.*

The geometrical parameters from Figure 3 can be determined as

$$d = r_t \cot\alpha, R_{2\mathrm{i}} = d + d_1, R_{2\mathrm{e}} = \frac{d+d_1}{\cos\alpha},$$
$$R_{3\mathrm{i}} = d + d_2\,, R_{3\mathrm{e}} = \frac{d+d_2}{\cos\alpha}. \tag{4.1}$$

Now, after determination of the resistance values for $R_2 = R_{2\mathrm{i}}$, $R_3 = R_{3\mathrm{i}}$ (labels as R_{gi}), i.e. for $R_2 = R_{2\mathrm{e}}$, $R_3 = R_{3\mathrm{e}}$ (labelled as R_{ge}), the approximate value of the grounding electrode resistance can be determined as the arithmetic mean

$$R_{\mathrm{g}} = \frac{R_{2\mathrm{e}} + R_{2\mathrm{i}}}{2} \text{ , i.e.} \tag{4.2a}$$

$$R_{\mathrm{g}} = \frac{1}{2\pi\sigma_1}\left(\frac{1}{r_1} - \frac{1}{r_{\mathrm{t}}}\right) + \frac{1}{2\pi\sigma_1(1-\cos\alpha)}\left(\frac{1}{r_{\mathrm{t}}+d} - \frac{R_{2\mathrm{e}}+R_{2\mathrm{i}}}{2R_{2\mathrm{e}}R_{2\mathrm{i}}}\right) +$$
$$+ \frac{1}{2\pi\sigma_2(1-\cos\alpha)}\left(\frac{R_{2\mathrm{e}}+R_{2\mathrm{i}}}{Rr_{2\mathrm{e}}R_{2\mathrm{i}}} - \frac{R_{3\mathrm{e}}+R_{3\mathrm{i}}}{2R_{3\mathrm{e}}R_{3\mathrm{i}}}\right) + \frac{1}{2\pi\sigma_3(1-\cos\alpha)}\frac{R_{3\mathrm{e}}+R_{3\mathrm{i}}}{2R_{3\mathrm{e}}R_{3\mathrm{i}}}. \tag{4.2b}$$

5 NUMERICAL RESULTS

The obtained results and relative errors obtained by the described method are shown in Table 1 for $\{\sigma_2/\sigma_1 = 5, \sigma_3/\sigma_1 = 10\}$ and Table 2 for $\{\sigma_2/\sigma_1 = 5, \sigma_3/\sigma_1 = 10\}$.

The set of other parameters` values is $\sigma_1 = 0.001\,\mathrm{S/m}$, $d_1 = 10\,\mathrm{m}$, $d_1 = 10\,\mathrm{m}$, $\alpha \in \{45^0, 50^0, 55^0, 60^0\}$ and $r_t \in \{10\,\mathrm{m}, 20\,\mathrm{m}\}$.

The values of the parameters have been selected based on *[11]*. The obtained results ($R_{g\,ap}$) were validated with the values obtained from the COMSOL program package application (R_g).

From the results shown in Tables 1-2, one can conclude that the relative error was not larger than 15,4%, which can be assumed as a satisfactory result. A more detailed analysis requires a more extensive data set, but at this research level, it can be indicated that the proposed simple approach can be assumed as valuable and applicable in practice.

Table 1: Grounding resistance for $\sigma_1 = 0.001\,\mathrm{S/m}$, $r_1 = 5\,\mathrm{m}$, $d_1 = 10\,\mathrm{m}$, $d_2 = 20\,\mathrm{m}$, $\sigma_2/\sigma_1 = 5$ *and* $\sigma_3/\sigma_1 = 10$

	α=45⁰		
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	22,2134	24,066	7,697997
20	23,5436	22,348	5,349919

	α=50⁰		
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	22,2324	23,735	6,330735
20	23,6881	22,327	6,096206

	α=55⁰		
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	22,3231	23,445	4,785242
20	23,8392	22,310	6,854325

	α=60⁰		
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	22,4740	23,198	3,120959
20	23,9916	22,295	7,609778

***Table 2:** Grounding resistance for* $\sigma_1 = 0.001\,\mathrm{S/m}$, $r_1 = 5\,\mathrm{m}$, $d_1 = 10\,\mathrm{m}$, $d_2 = 20\,\mathrm{m}$, $\sigma_2/\sigma_1 = 10$ *and* $\sigma_3/\sigma_1 = 50$.

α=45⁰			
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	20,3582	24,066	15,4068
20	22,3068	22,348	0,184357
α=50⁰			
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	20,6405	23,735	13,03771
20	22,5950	22,327	1,20034
α=55⁰			
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	20,9414	23,445	10,67861
20	22,8605	22,310	2,467503
α=60⁰			
r_t[m]	$R_{g\,ap}$ [Ω]	R_g[Ω]	Relative error [%]
10	21,2632	23,198	8,340374
20	23,1052	22,294	3,638647

6 CONCLUSIONS

The procedure, based on several recently proposed approaches for approximate determination of the resistance value of a hemispherical ground electrode placed at the top of the truncated cone, is presented in the paper. The obtained results indicate that this simple approach can be used for approximating the described ground non-homogeneity.

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