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OPTIMAL DISPERSED GENERATION PLACEMENT IN RADIAL DISTRIBUTION SYSTEMS USING A PATTERN OF LOAD FLOW PROCEDURE – CLUSTERING OPTIMISATION

OPTIMALNA POSTAVITEV RAZPRŠENIH VIROV ENERGIJE V RADIALNIH DISTRIBUCIJSKIH OMREŽJIH PO VZORCU DOLOČANJA PRETOKA MOČI – OPTIMIZACIJA GROZDENJA

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Abstract

Nowadays, distributed generation plays a vital role in distribution systems. It makes an indisputable contribution towards power loss minimisation and voltage profile improvement. To maximise the benefits of distributed generators, their location and size is of crucial importance. This paper describes the use of clustering optimisation (CO) as a highly effective method for determining the optimal placement and sizing of distributed generators. Assessment of the effectiveness from the clustering optimisation method is achieved by its demonstration on a 69-bus and 119-bus distribution systems. Furthermore, the results obtained from implementation of the proposed approach are compared to results from recent studies including analytical approaches, heuristic, and meta-heuristic, as well as mathematical programming algorithms. It is concluded that clustering optimisation is a simple and efficient method in terms of optimal allocation and sizing of distributed generators. It outweighs other approaches in terms of simplicity, results, and computation time.

Povzetek

Porazdeljena proizvodnja energije dandanes igra ključno vlogo v distribucijskih sistemih, saj nesporno prispeva k zmanjšanju izgub moči in izboljšanju napetostnega profila. Da bi povečali prednosti porazdeljenih virov, sta njihova lokacija in velikost ključnega pomena. Ta članek opisuje uporabo optimizacije združevanja v grozde (CO) kot zelo učinkovito metodo za določanje optimalne postavitve in dimenzioniranja porazdeljenih virov. Oceno učinkovitosti metode optimizacije grozdenja dosežemo z njeno demonstracijo na distribucijskih sistemih z 69 in 119 zbiralkami. Poleg tega rezultate, pridobljene z izvajanjem predlaganega pristopa, primerjamo z rezultati iz nedavnih študij, vključno z analitičnimi pristopi, hevristiko in meta-hevristiko ter algoritmi matematičnega programiranja. Na ta način ugotovimo, da je optimizacija združevanja v grozde preprosta in učinkovita metoda v smislu optimalne alokacije in dimenzioniranja porazdeljenih virov ter odtehta druge pristope v preprostosti, rezultatih in času izračuna.

1 INTRODUCTION

Increasing climate change, dwindling resources and greenhouse gas emissions have led to an increase in the exploitation of renewable energy sources for electricity production. As renewables are scattered around the country, their potential can mostly be tapped through integration to the distribution system as a form of distributed generation (DG). In recent years, the share of DGs in power systems has been significantly increasing. Distributed generation can be defined as any electricity generating technology, installed by the utility system or at the site of a utility customer, connected at the distribution system level of the electric grid. *[1]* Therefore, DG integration undoubtedly affects the power system, especially at the distribution level. Properly located and sized units have the potential to reduce total power losses in the system, improve the voltage profile, enhance reliability, enable greater available capacity for power transmission and reduce equipment stress. *[2]* On the other hand, if not optimally placed and sized, the installation of DGs in the grid could cause an increase of system losses, crippling of the voltage conditions, voltage flicker and an increased level of harmonics, all leading to considerably greater costs. *[3]* Hence, the use of an efficient optimisation method for sizing and placement is of immense importance to fully access the benefits of DGs. *[4-7]*

Numerous technologies can be considered as DGs. Photovoltaics and solar-thermal units, wind turbines, small hydro plants, geothermal units, all types of fuel cells and battery storage technologies can be categorised as DGs. *[8]* However, from a power system analysis point of view, any type of DG technology, depending on its specifics, can be presented as an active, reactive, or apparent power injection (PIN).

Over the past few years, a variety of different approaches and methods have been developed in relation to the issue of optimal sizing and allocation of PINs. They can generally be classified as analytical methods *[8-14]*, heuristic and meta-heuristic *[15-28]*, hybrid methods *[29-37]* and mathematical programming algorithms *[38-41]*. All approaches have unique advantages, but also have various limitations and setbacks to some extent.

Analytical techniques perform mathematical analysis of power distribution systems, resulting in a set of numerical equations that are used to formulate an objective function *[13,14]*. They have some remarkable features such as simplicity, easy implementation, and short computation time *[7,10]*. These techniques are suitable for variety of objectives, but they also have certain limitations. For instance, the authors in *[8]* present an analytical approach for determining the optimal location and size of a DG. The proposed algorithm consists of formulating a loss sensitivity factor based on equivalent current injection without requiring the Jacobian matrix, the admittance (or its inverse) matrix and manages to give competent results. However, the optimisation procedure only determines the location and size of a single DG unit that only injects active power. Another analytical method is presented in *[11]* which also considers only active power injection for the purpose of determining maximum power loss reduction.

Heuristic methods are mainly based on engineering experience and knowledge acquired through research as done in *[16]*. These methods aim to explore the search space in a particularly convenient manner. Heuristic methods are generally problem dependent. Their greatest setback, however, is that they always give a possible solution which is not necessarily optimal. For example, the heuristic method based on Uniform Voltage Distribution Algorithm presented in *[18]* despite being robust and fast, fails to give the optimal allocation of reactive power injections.

Meta-heuristic approaches (also known as population-based optimisation methods) are widely used because of their problem-independence and ability to provide competent results. However, most meta-heuristic algorithms are only approximation algorithms as they cannot always find the global optimal solution. Furthermore, they require tuning of a great number of parameters as well as long computation time due to their iteration-based nature. Teaching learning-based optimisation - TLBO *[42]* algorithm overcomes the problem of defining algorithm specific parameters. The optimality of the results and convergence properties of the TLBO algorithm have been improved in *[42]* by introducing a new quasi-oppositional TLBO - QOTLBO method. The QOTLBO algorithm; however, is parameter-dependent and has the tendency to get trapped in a local optimum. These shortcomings have been overcome with the comprehensive TLBO - CTLBO method presented in *[43]*. Nevertheless, all these methods can be classified as meta-heuristic approaches, hence retaining the advantages and disadvantages from this group of methods.

Combining methods from different groups, i.e., hybrid methods can contribute to overcoming some of the shortcomings from the individual methods. Therefore, the authors in *[29]* present a hybrid analytical and meta-heuristic method for optimal placement and sizing of multiple DGs of different types. DG sizes are determined by analytical method, whereas their locations by a PSO algorithm. The results obtained from this method are superior compared to results attained from PSO and analytical methods. However, due to the iteration-based nature of the PSO method, the algorithm can encounter difficulties in terms of convergence speed and accuracy by increasing the number of variables and implementation on large scale networks. LSF-BFOA method is presented in *[31]* and used for simultaneous placement of DG and DSTATCOM. Location of the DG and DSTATCOM is determined by using a loss sensitivity factor - LSF and the optimal size is then determined by using a Bacterial Foraging Optimisation Algorithm - BFOA. Despite offering superior results, the LSF-BFOA method is characterised with a relatively long computation time. Something similar is done in *[35]* using loss sensitivity factors - LSF and then gradually placing DGs in a set determined from the sensitivity analysis.

Table 1: *Overview and number of methods the proposed approach is compared with*

Method	Citation	Approach	№ of methods citation outperforms	Test case	Year
AM	[8]	analytical	2	IEEE-69	2009
IA	[10]	analytical	2	IEEE-69	2013
AA	[13]	analytical	2	IEEE-69	2015
EA	[14]	analytical	5	IEEE-69	2016
NH	[15]	heuristic	5	IEEE-69&119	2019
BPSO-SLFA	[26]	heuristic	3	IEEE-69	2020
TLBO	[42]	meta-heuristic	4	IEEE-69	2014
QOTLBO	[42]	meta-heuristic	4	IEEE-69&119	2014
IWO	[27]	meta-heuristic	5	IEEE-69	2016
CTLBO	[43]	meta-heuristic	2	IEEE-69&119	2018
DE-TLCHS	[25]	meta-heuristic	1	IEEE-69	2019
MRFO	[28]	meta-heuristic	8	IEEE-69	2020
HPSO	[29]	hybrid	2	IEEE-69	2014
LSF-BFOA	[31]	hybrid	3	IEEE-119	2016
MFO	[37]	hybrid	/	IEEE-69	2017
BIBC	[36]	hybrid	/	IEEE-69	2018
QODELFA	[32]	hybrid	6	IEEE-69&119	2019
LSF-SSA	[33]	hybrid	2	IEEE-69	2019
DGPI	[35]	hybrid	6	IEEE-69	2020
FP-PSO	[34]	hybrid	/	IEEE-69	2020
NLP-PLS	[40]	mathematical	8	IEEE-69	2016
MISOCP	[41]	mathematical	8	IEEE-69&119	2019

Real-world problem mathematical formulations are derived under certain assumptions that should reflect and model the problems physical behaviour in as much detail as possible. Even with these assumptions, the solution to large scale power systems is not simple. Methods that propose mathematical programming algorithms are usually unsuitable for large power systems and prone to errors when linearising its non-linear characteristics and uncertainties. It is desirable that a solution to any problem should be a global optimum. However, solutions obtained by mathematical optimisation are not necessarily globally optimal. The reason behind this is usually the modelling complexity and linearisation processes which can be quite challenging. *[39]* These facts make it difficult for these methods to deal effectively with many power system problems through strict mathematical formulation alone. Furthermore, gradient search, linear programming, dynamic programming, sequential quadratic programming, and nonlinear programming are considered as traditional methods, but none of them provides a solution to complex problem or optimal solution within a reasonable time. *[7]* It should be mentioned; however, that if the mathematical formulations are reflecting the problems behaviour exactly, even though computation time is rather long, mathematical programming algorithms guarantee a highly superior if not globally optimal result. *[41]*

This paper outlines a new approach to solving the problem of optimal allocation and sizing of PINs by presenting a simple search-based algorithm that in its essence is a pattern of using a load flow procedure. The main motivation behind this research is to introduce a simple solution to a highly complex non-linear problem. By iteratively looping through the network buses, the algorithm places a PIN of a defined size and type (active, reactive, or apparent) at specific locations yielding minimum power losses while keeping all voltages at their acceptable levels. Moreover, the optimal power factor is also determined in the case of apparent type of PIN. The proposed method is demonstrated on a 69-bus and 119-bus distribution system. Table 1 presents an overview of methods from all mentioned groups of optimisation approaches over the past decade, along with several methods/approaches they outperform. Results from the proposed method are compared to those from Tab. 1. It can be concluded that the proposed method is superior to every recent methodology, apart from a very few exceptions. It produces repetitive and unique results; it is extremely easy to implement, and it has a short computation time.

2 PROBLEM FORMULATION

Determining the optimal location and size of a PIN in the distribution system is a complex non-linear problem. As previously mentioned, for a certain location (bus) in the network, as the size of the PIN increases, an adequate decrease in losses can be noticed. However, after exceeding a certain PIN threshold, the losses start to increase again due to a reverse power flow towards the slack/supply bus. The size of the PIN should at most be such that it is consumable within the distribution substation limits. *[12]*

The problem of determining the optimal PIN size and locations is quantified through objective function that can take any mathematical formulation in terms of complexity. For purposes of comparison to other relevant research, objective function introduced in this paper includes costs for energy and power losses only and should yield a least possible value, i.e., minimum:

$$\min F = c_e \cdot \Delta W + c_p \cdot \Delta P_{max} \tag{2.1}$$

with c_e being the cost for kilowatt-hour of electricity ($/kWh), ΔW being the electricity losses within the observed period (kWh), c_p being the cost per kilowatt for reduction in losses ($/kW) and ΔP_{max} being the maximum power losses during the same period (kW). To determine the minimum value of the objective function, it is necessary to obtain the minimum value of power losses which would also result in a minimum value of electricity losses during the observed period.

The number of constraints and their formulation can be regarded as a way of guiding the optimisation algorithm towards a feasible and possibly optimal solution because not necessarily always these two go hand in hand. Imposing a high number of constraints generally narrows the optimisation algorithm search path and while providing a more accurate and realistic approach, the trade-off is a feasible but not necessarily optimal solution and vice versa. For purposes of comparison to other relevant research, during all calculations, constrains on bus voltage values are imposed by setting an upper and lower bound, *[44]* resulting in the following constraint:

$$V_{min} \le V_i \le V_{max}, \text{ for } i = 1, \ldots, NB \tag{2.2}$$

where V_{min}= 0.9 pu, V_{max}= 1.1 pu and NB being the number of buses in the given network.

3 LOAD FLOW

Aiming for optimal placement and sizing of PIN, it is first and foremost indispensable to determine the parameters of interest in the distribution system, i.e., bus voltages and power losses. Due to distribution systems specific nature such as high R/X ratio and radial structure, conventional load flow methods usually fail to give satisfactory results. *[45]* Of all proposed power flow solution methods for distribution systems, backward/forward sweep methods are most widely used because of their computational efficiency and robust convergence characteristics. *[46-48]* The efficiency of the sweeps can be enhanced with oriented branch numbering *[49]*, the only requirement being that the sending bus number i is smaller than the receiving bus number k, i.e., i < k (Fig. 1). Indices from the branch's sending buses are stored in a vector f, such that i = f(k), where k is the index of the branch's receiving bus. Additionally, by introducing a fictitious branch with index 1 (sending end index 0), the number of branches NL becomes equal to the number of buses NB, making the sweep procedure very simple and efficient.

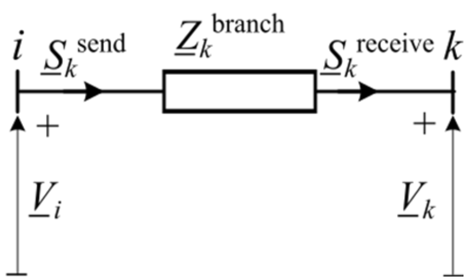

Figure 1: *Branch representation: branch k between buses i (sending end) and k (receiving end)*

Backward sweep consists of equations that calculate the power flow through branches starting from the last one and proceeding in a backward direction towards the supply/slack bus. First, receiving end branch flows are calculated using (3.1), where $\underline{S}_{demand}^{receive}$ is the power demand at the branch's receiving end and $\underline{Y}_k^{shunt}$ is shunt admittance connected to bus k due to capacitance of the lines and/or connected capacitors (3.2).

$$\underline{S}_k^{receive} = \underline{S}_{demand}^{receive} + (\underline{Y}_k^{shunt})^* \cdot V_k^2, \text{ for } k = 1\ldots, NB \tag{3.1}$$

$$\underline{Y}_k^{shunt} = j(B_k^{lines} + B_k^{cap}), \text{ for } k = 1\ldots, NB \tag{3.2}$$

$$\underline{S}_{demand,new}^{receive} = \underline{S}_{demand}^{receive} - \underline{S}_k^{PIN}, \text{ for } k = 1\ldots, NB \tag{3.3}$$

Afterwards, sending end branch flows are calculated using (3.4) and added to the branch's receiving end that supplies them (3.5).

$$\underline{S}_k^{send} = \underline{S}_k^{receive} + \underline{Z}_k^{branch} \left| \frac{\underline{S}_k^{receive}}{\underline{V}_k} \right|^2, \text{ for } k = NL, NL-1\ldots, 2 \tag{3.4}$$

$$\underline{S}_{i,new}^{receive} = \underline{S}_i^{receive} + \underline{S}_k^{send}, \text{ for } k = NL, NL-1\ldots, 2 \qquad 3.5)$$

Forward sweep is performed to determine the voltage drops and actual voltages of each bus starting from the slack bus and proceeding in the forward direction towards the last bus using (3.6).

$$\underline{V}_k = \underline{V}_i - \underline{Z}_k^{branch} \cdot \left(\frac{\underline{S}_k^{send}}{\underline{V}_i} \right)^*, \text{ for } k = 2, \ldots, NL \tag{3.6}$$

After completing a sweep, the calculated voltages of the present iteration are compared to those from the previous one. If the voltage mismatch between two consecutive iterations is less than the specified tolerance of ε = 10^{-4}, convergence can be achieved. Otherwise, the procedure is repeated until convergence of the solution is attained. After determining the power flow through the branches, it is easy to calculate the active power losses by simply subtracting the real parts from the complex sending and receiving end branch power flows (3.7):

$$P_k^{receive} \tag{3.7}$$

where $P_k^{receive}$ is active power from the branch's sending bus and $P_k^{receive}$ is active power from the branch's receiving bus.

4 CLUSTERING OPTIMISATION

As has been previously mentioned, many approaches have been proposed in terms of determining the best possible location and size of PINs in an existing distribution system. However, some of them only consider PINs with unity power factor, some are applicable only for single power injecting unit allocation, others offer nearly optimal solutions, and some have inconveniently high computation time. To overcome these shortcomings, a simple search-based clustering optimisation (CO) is presented in this paper.

Basic idea behind the CO is by iteratively probing all buses from the distribution system apart from the slack bus, to place PINs of user-defined size and number of locations at locations that would yield least possible losses, which is quantified through (2.1). Power injections can be considered as purely active, purely reactive, or apparent. Iterative bus probing eliminates the need of using any kind of sensitivity analysis for bus selection as the process itself implicitly does that.

In case of apparent power injection, the optimal value of the power factor is also determined. Proposed approach does not constrain the power factor value when searching for its optimal due to several reasons: *[4-7]*

- Most if not all utility owners demand from their dispersed generation PINs reactive power support without any specified value or quantity;
- Most dispersed generation PINs are owned by private entities which are usually if not exclusively more incentivised when injecting purely active power, i.e., they tend to operate at unity power factor. In some cases, they're required to operate at unity power factor at their point of common coupling, i.e., they should only produce reactive power for their own personal operational requirements;
- Imposing constraints on power factor value implicitly disables the proposed approach in reaching competitive and comparable results to other relevant methods.

During the CO procedure, all buses apart from the slack are considered candidate buses for optimal PIN placement, i.e., there is no bus selection procedure nor weak bus sensitivity analysis. There are many reasons for avoiding this type of bus pre-processing. Not all buses possess the same sensitivity trend when subjected to a same rate of power injection change. This is owed to network topology, load, and voltage profile. For example, if a single small power injection is placed on every bus successively, they will attain a certain sensitivity index based on the applied analysis. This index can be used to rank the buses in terms of their susceptibility towards receiving that particular power injection. However, adding a successive power injection of same size introduces a shift in bus sensitivity index following its appropriate trend that is unknown, meaning there may be a shift in position from the previous bus ranking. For a fixed number of locations for PIN placement this is a huge problem since the set of locations potentially changes by successively adding more power injections per bus and the analysis becomes obsolete. There are ways one can alleviate that, i.e., sensitivity analysis and bus ranking can be performed for base case scenario and the obtained set of buses can be kept constant throughout the process of optimisation. However, one should bear in mind that different sensitivity analysis imposes different set of candidate buses and consequently a different solution that may or may not be optimal. *[50]*

The proposed approach alleviates this drawback by probing each bus with a small power injection, i.e., implicitly choosing locations that yield least possible losses while inherently following the buses sensitivity trend, hence avoiding the bus selection procedure. The CO algorithm calculates power losses in the distribution network using a linearised (flat start, single iteration) load flow *[48]* while keeping the voltage profile of the system within prescribed limits. The use of linearised load flow implies that load flow procedure finishes after one iteration, i.e., the resulting power losses and voltages are obtained after a single iteration of backward/forward sweeps. The latter enables placing PINs at each bus successively while keeping the computation time for the entire procedure significantly short.

Values of power losses obtained from a linear load flow are smaller compared to those obtained from regular iterative load flow due to flat start, i.e., all bus voltages are equal to the distribution system nominal voltage. Therefore, to obtain the actual values of power losses and voltages, another non-linear load flow is performed at the end of the clustering procedure. This approach is legitimate as the difference in values obtained from both linear and non-linear load flow solution is equal for each bus, therefore the optimal PIN size and their optimal locations do not change.

The number of locations for PIN placement, type, size and φ_{step} (in case of apparent PIN for attaining optimal power factor) are all user-defined. The CO algorithm performs through the following steps:

- Step 1. Read distribution system line and load data and obtain total number of buses N_{loc}. Initialise user-defined PIN size and type, desired number of locations for PIN placement N_{loc}, power factor angle step $n = 0$ and counter $n = 0$ (current number of buses where PINs are placed). Perform base case load flow and obtain values for distribution system power losses ΔP_0 and minimum voltage $U_{\min,0}$.
- Step 2. Place PIN units with predefined size $\left(\underline{S}_{unit}^{PIN}\right)$ and type (active, reactive, or apparent) at each bus φ_{step}. Perform subsequent linearised load flows (flat start, single iteration) for each PIN at each bus i, to check for power loss reduction. If apparent PIN type is considered, perform additional linearised load flows for each value of φ_{step} to check for optimal power factor as well, for each $-\pi / 2$ at each bus. The value of $-\pi / 2$ goes from $-\pi / 2$ to $+\pi / 2$ with a resolution of $\underline{S}_i^{PIN}$. Stop placing PINs per bus i if no loss reduction and no voltage improvement is achieved. Store the cumulative PIN for bus i as $\underline{S}_i^{PIN}$ and its appropriate losses ΔP_i and minimum voltage $V_i^{\min}$.
- Step 3. Using the results from Step 2, for the set of buses $i = 2, 3, \ldots N_B$, find bus m that ensures minimum active power losses $\Delta P_m = \min(\Delta P_2, \Delta P_3, \ldots, \Delta P_{N_B})$. Place the power injection at $\underline{S}_m^{PIN}$ location m and increase the counter n by 1.

- Step 4. Update the power demand with $n = N_{loc}$ at the target bus m obtained from Step 3 using (3.3) and repeat the previous two steps until the desired number of locations is achieved ($n = N_{loc}$) or no power loss reduction is detected.
- Step 5. Perform final non-linear load flow for the cumulative PINs and placement locations to obtain actual values for power losses and voltages.

Figure 2 represents a flow chart of the CO algorithm to further support and explain its performance. Additionally, an example for illustrative purposes is presented below.

Example: The CO algorithm is illustrated on a 12.66 kV, 69-bus distribution system searching a single location by placing apparent PINs of size φ kVA. The value of φ goes from $+\pi/2$ to $+\pi/2$ with $\varphi_{step} = \pi/9$. The main purpose is power loss minimisation, i.e., minimum value of (2.1) with $\Delta P_0 = 225$ and $\Delta P_0 = 225$. Base case losses are $\Delta P_0 = 225$ kW and lowest voltage occurs at bus 65, i.e., $U_{min,0}^{@65} = 0.9092$ pu. All buses apart from the slack bus are considered for potential PIN placement.
Considering the use of apparent PIN in this example, the procedure also determines the optimal value of $\varphi_{step} = \pi/9$ which yields minimum losses, by performing subsequent linearised load flows for each value of $\varphi_{step} = \pi/9$ from $+\pi/2$ to $+\pi/2$. The latter is done for each $\underline{S}_{unit}^{PIN}$ per bus. The CO builds the cluster by continuously adding φ per bus and obtaining the optimal value of φ until no further loss reduction is detected. When no further power loss reduction is detected by adding additional $\underline{S}_i^{PIN}$ at a certain bus, the cluster, i.e., cumulative PIN with size $\underline{S}_i^{PIN}$ and cumulative value of φ is formed. Figure 3 shows a normalised histogram of PINs and power losses for every candidate bus. Losses are normalised to ΔP_0 and PINs are normalised to the maximum system injection per bus, which for this example is 4944.7 kVA.

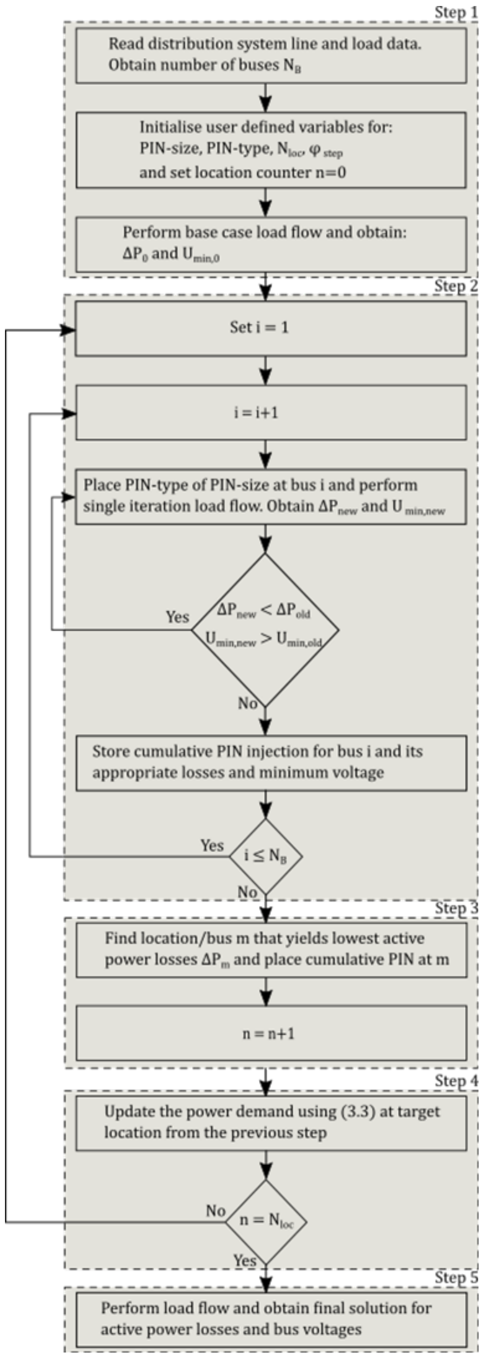

Figure 2: *Flow chart of the CO algorithm*

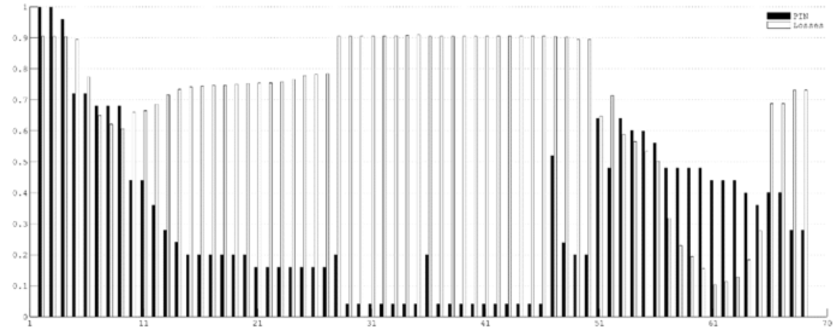

Figure 3: *Normalised histograms of PINs and power losses for all candidate buses from the 69-bus distribution system*

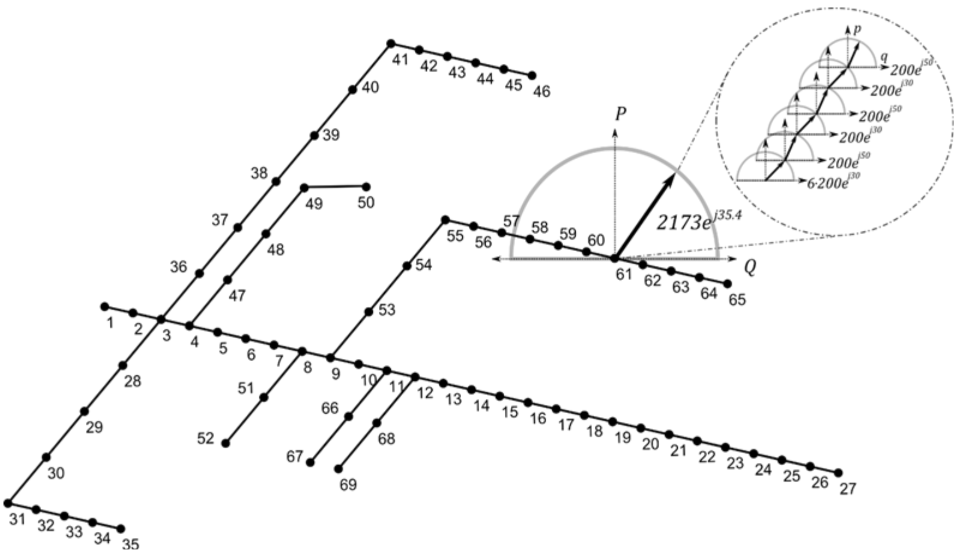

Figure 4: *Cluster formation at bus 61 from the 69-bus distribution system*

Once the cumulative PIN for every bus ($\underline{S}_{@bus}^{PIN}$) is determined along with its appropriate losses, the CO performs a simple search for minimum losses and optimal bus for PIN placement. The cumulative PIN is then placed at the determined/optimal location. In this example, the optimal location is bus 61 (Fig. 3), while the cumulative PIN value is $\underline{S}_{@61}^{PIN} = 2173.5 \cdot e^{j35.42^o}$ kVA.

The cluster formation for the optimal location is visually presented in Fig. 4 and Tab. 2.

Table 2: Cluster formation at bus 61 from the 69-bus distribution system

Iteration №	$\underline{S}_{unit}^{PIN}$	φ
1-6	$200e^{j30}$	30^0
7	$200e^{j50}$	50^0
8	$200e^{j30}$	30^0
9	$200e^{j50}$	50^0
10	$200e^{j30}$	30^0
11	$200e^{j50}$	50^0
$\underline{S}_{@61}^{PIN}$	$2173e^{j35.42}$	35.42^0

Step 4 is omitted in this example since there is only one potential location for PIN placement. However, the updating of power demand at bus 61 ($\underline{S}_{demand,new}^{@61} = \underline{S}_{demand}^{@61} - \underline{S}_{@61}^{PIN}$) is still performed considering the new condition in the distribution system.

Next, the CO performs a final non-linear load flow to determine the actual power losses $\Delta P = 23.3370$ kW and minimum voltage $U_{min}^{@27} = 0.9720$ pu in the distribution system.

5 ROBUSTNESS, CONVERGENCE PROPERTIES AND COMPUTATION TIME

The proposed CO method possesses several remarkable features, making it superior in terms of complexity/simplicity, implementation, and results compared to other existing search-orientated methods (referenced in Section 1) used for optimal power injection sizing and allocation.

- There is no bus selection procedure, nor any form of mathematical modelling and analysis for bus selection. The CO method implicitly chooses locations/buses based on a simple search described in Section 4;
- The CO method produces unique, mostly superior, repetitive, and easily reproducible results compared to other search-orientated methods;
- The algorithm operates in relatively short computation time that is dependent on distribution system size and PIN type/size. In case of apparent PINs, computation time is generally longer because optimal power factor is also being determined. Moreover, smaller size of PIN units engenders longer computation time due to an increased number of performed calculations per bus;
- Users are disburdened from complex mathematical formulations or models as in its essence, the CO method is a 'pattern of using a load flow procedure' making it extremely easy to implement.

Table 3 summarises the CO's performance for the 69-bus distribution system. It can be noted that for rather large range of PIN sizes the power loss and cumulative PIN deviations are small. This is owed to the fact that cluster formation with a certain PIN size can be considered as cluster resolution. Larger PIN size imposes faster and rougher cluster formation, i.e., shorter computation time. Smaller PIN size imposes slower and smoother cluster formation but at the expense of longer computation time. Furthermore, PIN size and type are user-defined variables which can be considered a drawback if one's so insistent on that tenth or hundredth of a kilowatt losses delta. Real-life distribution system line and load data acquisition can be quite a difficult task. Additionally, distribution system data collection can introduce errors of magnitude in several tens of percent so it can be safely said that the method is as accurate as the accuracy of the input data. Tab. 3 should only serve for user's illustrative and indicative purposes when choosing PIN size regarding output results and computation time.

The CO algorithm is implemented in MATLAB R2018a. All calculations are performed on a laptop configuration with a 2.6-GHz Intel Core i5-4210M processor with 8Gb of RAM. All solutions from references subject to comparison with the CO algorithm are ran through a load flow procedure to check for accuracy of the results. Figure 5 illustrates convergence curves for the CO algorithm when searching for a single location and three locations accordingly. It should be noted that the ending point of the single location convergence curve is a starting point for the next one etc. The latter is owed to the fact that CO algorithm successively chooses buses (one by one) for PIN placement which is explained in detail in Section 4.

Table 3: *Power loss deviation and calculation time depending on PIN size, type, and number of locations for the 69-bus distribution system*

№	PIN Size (kVA)	Active PIN ΔP_P (kW)	Active PIN t_P (sec.)	Active PIN Total (kW)	Reactive PIN ΔP_Q (kW)	Reactive PIN t_Q (sec.)	Reactive PIN Total (kVAr)	Apparent PIN ΔP_S (kW)	Apparent PIN t_S (sec.)	Apparent PIN Total (kVA)
1	1	83.26	8.69	1841	152.06	6.01	1307	23.17	113.13	$2237.65e^{j35.45}$
	50	83.24	0.20	1850	152.08	0.13	1300	23.19	2.23	$2223.31e^{j35.30}$
	100	83.41	0.09	1800	152.08	0.07	1300	23.2	1.15	$2273.10e^{j35.18}$
	200	83.41	0.06	1800	153.43	0.04	1200	23.34	0.60	$2173.52e^{j35.42}$
2	1	71.8	12.24	2353	146.49	8.34	1654	7.57	149.32	$2851.33e^{j35.16}$
	50	71.85	0.25	2350	146.51	0.23	1650	7.51	3.00	$2797.55e^{j34.27}$
	100	71.83	0.19	2300	146.61	0.11	1600	7.88	1.62	$2867.80e^{j34.79}$
	200	72.42	0.10	2300	147.78	0.06	1600	7.5	0.91	$2765.32e^{j35.68}$
3	1	70.28	13.39	3072	145.70	9.99	2168	5.29	159.29	$3734.45e^{j35.24}$
	50	70.34	0.29	3050	145.72	0.25	2150	5.22	3.67	$3706.98e^{j35.03}$
	100	70.32	0.17	3000	145.83	0.17	2100	5.59	1.87	$3755.30e^{j35.23}$
	200	69.78	0.10	2600	146.99	0.10	2200	5.24	1.41	$3556.18e^{j35.52}$

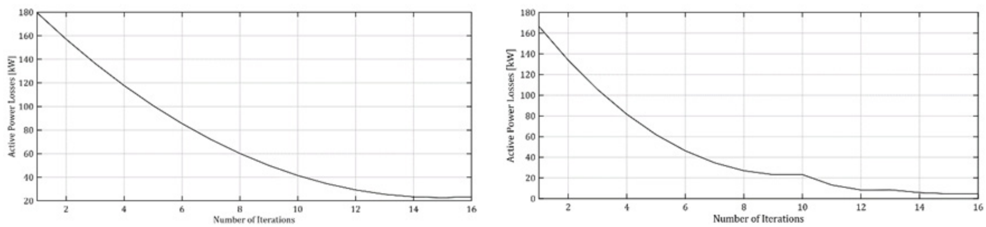

Figure 5: *Convergence curves for power losses of the CO method: a) single location 69-bus, b) three locations 69-bus*

6 CASE STUDIES

Assessment of the effectiveness of the proposed CO method is achieved by its application on two distribution systems: 12.66 kV, 69-bus distribution system [51] and an 11 kV, 119-bus distribution system. [52] To emphasise the unique properties of the CO method, results obtained from the two distribution systems are compared to results from recent work. For purposes of comparison, data concerning the objective function (1) is given with $c_p = 1$ and $c_p = 1$. Otherwise, objective function values in Tab. 4 and Tab. 6 are calculated with $c_e = 0.067$ \$/kWh and $c_p = 16$ \$/kW.

Base-case values for power losses, minimum voltages, and objective function (1) for both systems are:

69-bus. $U_{\min} = 0.9092$ (kW), $U_{\min} = 0.9092$ (pu) and $F_0 = 135657$ (\$/year)

119-bus. $\Delta P_0 = 1298$ (kW), $F_0 = 782590$ (pu) and $F_0 = 782590$ (\$/year)

6.1 69-bus distribution system

Table 4 shows results from the performed analysis using the proposed CO method on the 69-bus distribution system. Nine different scenarios are considered, i.e., three scenarios for each PIN type (active – P, reactive – Q and apparent – S) for one, two and three locations. For each scenario, minimum voltages and computation time is also presented. Table 4 suggests that the CO method performs remarkably well presenting unique results in terms of power losses, minimum voltage, and computation time.

Table 4: *Results from CO for 69-bus distribution system (power injection size is expressed in P(kW), Q(kVAr) and S(kVA); voltages in pu)*

PIN № & Type	Size@Bus			U_{min}@Bus	ΔP (kW)	F ($/ year)	t (sec)
1P	1869.3@61			0.9683@27	83.22	50175	0.14
2P	1836@61	516@17		0.9807@65	71.77	43272	0.81
3P	1755@61	390@21	390@11	0.9795@65	69.7	42024	0.09
1Q	1313@61			0.9305@65	152.05	91674	0.38
2Q	1296@61	336@17		0.9314@65	146.48	88316	0.16
3Q	1302@61	350@17	518@50	0.9315@65	145.68	87833	0.57
1S	$2249.5e^{j35.33}$@61			0.9725@27	23.17	13970	2.22
2S	$2209.5e^{j35.29}$@61	$649.6e^{j34.00}$@17		0.9943@50	7.44	4486	3.64
3S	$2120.5e^{j35.55}$@61	$471.4e^{j35.00}$@21	$471.4e^{j35.00}$@11	0.9973@50	4.6	2773	2.90

Table 5: *Table of comparison for 69-bus distribution system*

Method	Approach	Scenario	ΔP (kW)	t (sec.)	CO outperforms
AM *[8]*	analytical	1P	92.00	0.078	Yes
IA *[10]*	analytical	3P	69.96	0.71	Yes
		3S	12.72	/	Yes
AA *[13]*	analytical	3P	70.20	/	Yes
		3S	5.92	/	Yes
EA *[14]*	analytical	1P	83.23	0.2	Yes
		1S	23.26	/	Yes
NH *[15]*	heuristic	3P	69.70	/	Yes
		3Q	145.30	/	No
BPSO-SLFA *[26]*	heuristic	1P	152.15	/	Yes
TLBO *[42]*	meta-heuristic	3P	72.40	/	Yes
QOTLBO *[42]*	meta-heuristic	3P	71.62	0.078	Yes
IWO *[27]*	meta-heuristic	3P	74.59	5.7	Yes
		3S	13.64	/	Yes
CTLBO *[43]*	meta-heuristic	3P	69.43	/	No
DE-TLCHS *[25]*	meta-heuristic	3S	5.00	/	Yes
MRFO *[28]*	meta-heuristic	3P	69.43	/	No

HPSO *[29]*	hybrid	1P	83.37	/	Yes
		3S	4.61	/	Yes
MFO *[37]*	hybrid	1P	83.22	/	Yes
		2S	9.47	/	Yes
BIBC *[36]*	hybrid	3P	91.54	/	Yes
QODELFA *[32]*	hybrid	3P	69.43	/	No
		3S	12.07	/	Yes
LSF-SSA *[33]*	hybrid	1P	83.17	/	No
		1S	27.91	/	Yes
DGPI *[35]*	hybrid	1P	83.22	0.24	Yes
		1S	23.30	0.28	Yes
FP-PSO *[34]*	hybrid	1S	61.67	/	Yes
		2S	43.67	/	Yes
NLP-PLS *[40]*	mathematical	1P	83.23	0.078	Yes
		1S	23.18	0.078	Yes
MISOCP *[41]*	mathematical	3P	69.43	/	No
		3S	4.27	/	No

Table 5 compares the results from the CO method to other methods that use various approaches. Comparison is made for the simplest and the most complicated case presented in the cited references. All results are run through a load flow program to check for accuracy and adequate comparison. CO outperforms in almost every case. It is important to bear in mind that power loss differences per scenario, amongst all compared methods, differ within tenth of a kilowatt at best and several kilowatts at worst, regardless the method and its approach. Given the uncertainty of real distribution systems input data, it can be said that methods accuracy strongly depends on the accuracy of the input data. This is where CO strongly outperforms any other method as it is extremely simple for implementation - pattern of using a load flow procedure, something that every power system engineer knows extremely well. Not everyone possesses a strong modelling and mathematical knowledge and background to shape these types of problems, but certainly everyone or almost everyone knows how to implement a load flow procedure and do a simple search. In terms of computation time (wherever noted, '/' in Table 5 means unreported), CO outperforms most of the methods. Mathematical methods depending on the problem size could take several tens of minutes or even hours in some cases. *[41]* Analytic, heuristic and hybrid methods possess computation time comparable to the CO method.

6.2 119-bus distribution system

Performance evaluation of the CO method for the 119-bus distribution system is attained through five scenarios including: three scenarios for optimal placement and sizing of active, reactive, and apparent PINs at five locations, and two more scenarios including optimal placement and sizing of active and apparent PINs with optimal power factor at seven locations.

Table 6 shows results from the performed analysis using the CO method on the 119-bus distribution system. Comparison results are presented in Tab. 7. Again, all results are run through a load flow program to check for accuracy and adequate comparison. CO outperforms in almost every case, except when compared to MISOCP. The reason behind this is that in *[41]*, an exact model for the distribution system and load flow equations is used with no linearisation and no neglections, meaning that the obtained results present a global optimum. This is a mathematical approach and takes quite a while to reach an optimal solution (several minutes to several hours depending on the scenario). However, the CO method gives results that are very near those presented with MISOCP and in reasonable computation time as it can be seen from Tab. 6. Compared to other methods, CO outperforms in terms of power losses, i.e., solutions differ from several kilowatts at best to several tens of kilowatts at worst in favour of the proposed algorithm. Computation time is better, if not comparable, to other methods subject to comparison.

Table 6: *Results from CO for 119-bus distribution system (power injection size is expressed in P(kW), Q(kVAr) and S(kVA); voltages in pu)*

	Size@Bus				
PIN № & Type	5P	5Q	5S	7P	**7S**
	2800@50	4400@29	$3942e^{j42.0}$@50	1760@20	$2072e^{j36.6}$@20
	2800@71	2600@50	$3179e^{j32.5}$@72	1870@41	$2420e^{j35.7}$@41
	2400@79	1800@72	$2765e^{j35.7}$@80	2860@50	$3796e^{j42.8}$@50
	1600@96	1700@80	$1981e^{j34.0}$@96	2860@71	$3131e^{j32.2}$@72
	2800@110	2300@110	$3546e^{j38.9}$@110	2200@80	$2760e^{j37.5}$@80
				1540@97	$2082e^{j33.3}$@96
				3080@109	$3447e^{j40.0}$@110
Total PIN (kVA)	12400	12800	15413	16170	19708
Δ*P* (kW)	574.33	857.69	211.58	515.82	127.61
U_{min} (pu)	0.955	0.9074	0.9608	0.9582	0.9764
***F* ($/year)**	346280	517120	127570	311000	76940
***t* (sec)**	0.82	2.14	9.42	3.13	13.08

Table 7: *Table of comparison for 119-bus distribution system*

Method	**Approach**	**Scenario**	**ΔP (kW)**	**t (sec)**	**CO outperforms**
NH *[15]*	heuristic	5P	580.74	2.5	Yes
		5Q	861.60	2.0	Yes
		5S	227.50	5.4	Yes
		7P	515.70	4.1	No
		7S	128.80	6.2	Yes
QOTLBO *[42]*	meta-heuristic	7P	576.00	20.83	Yes

CTLBO *[43]*	meta-heuristic	7P	516.25	/	Yes
LSF-BFOA *[31]*	hybrid	5P	578.97	23.24	Yes
		5Q	871.40	23.21	Yes
		5S	227.90	24.65	Yes
		7P	526.34	24.96	Yes
		7S	132.10	25.96	Yes
QODELFA *[32]*	hybrid	7P	518.65	/	Yes
		7S	132.79	/	Yes
MISOCP *[41]*	mathematical	5P	571.29	/	No
		5Q	856.37	/	No
		5S	208.13	/	No
		7P	513.27	/	No
		7S	123.37	/	No

7 CONCLUSION

This paper presents a clustering optimisation (CO) method for optimal placement and sizing of DG's presented as active, reactive, or apparent power injections (PINs) with the aim of obtaining minimum power losses while maintaining an acceptable system voltage profile. The effectiveness of the proposed method is assessed through simulations performed on a 69-bus and 119-bus distribution systems. It can be concluded that the proposed method performs outstandingly, presenting unique results in all considered scenarios. Results attained from the performed analysis are compared to results from recent methodologies stretching back over the past decade. Tables 5 and 7, coupled with the information from Tab. 1, suggest that the proposed method is implicitly compared to over 50 other methods. It is noted that for most scenarios, CO gives lower power losses and total PIN size compared to other methods, while for other scenarios it presents slightly higher power losses and total PIN size. Power losses and total PIN sizes attained from the CO method are of the same order of magnitude as power losses and total PIN sizes attained from recent methodologies in all considered cases. Therefore, the slight deviation (fractions of kilowatts) in values obtained from the CO method compared to other methodologies is practically insignificant/negligible, considering the uncertainty of input data.

In its essence, the CO method is a pattern of using a load flow procedure. It does not include solving complex mathematical formulations, and it does not operate with population, nor its creation and iterative updating. In addition to this, it does not use problem coding and/or solution decoding and has no convergence problems, the latter of which renders the method superior in terms of its simplicity/complexity and easy implementation. Furthermore, the lack of laborious procedures makes the method significantly intelligible, simple, and easy to reproduce. Since the methods working principle is based on simple mathematical formulations and there is no dedicated bus selection procedure, optimal or nearly optimal solutions are obtained remarkably quickly. Short computation time enhances the convenience of using this method. Finally, another salient advantage of the proposed method is the recurrence of results, i.e., the results obtained are repetitive from simulation to simulation, which is not the case for some of the other existing methods.

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